# $\alpha$ – العادية وبعض تطبيقاتها في الفضاءات الضبابية الحدسية

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**الملخص العربي :**

الهدف من هذا البحث هو دراسة الفصل الضبابي الحدسي العادي من النوع – $\alpha$ وذلك بدراسة الصور من الدوال المستمرة الضبابية الحدسية كذلك سوف ندرس الفصل الضبابي الحدسي العادي من النوع - $\pi g\alpha$ ودراسة خاصية - $\pi g\alpha$ العادية في الفضاءات الجزئية, علاوة على ذلك سوف نناقش بعض خواصها.

## On $\alpha$ -Normal And Some Applactions In Intuitionistic Fuzzy Spaces

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***Abstract***

The aim of this paper is to study the class of intuitionistic fuzzy $\alpha$ -normal spaces with studying the forms of intuitionistic fuzzy continuous functions. Also we study the class of intuitionistic fuzzy $\pi g\alpha$ - normal, and $\pi g\alpha$-normality in subspaces. Moreover, we investigate some of their properties.

***Keywords:***

intuitionistic fuzzy $\alpha$ -open set, intuitionistic fuzzy $\pi g\alpha$ - closed set,
intuitionistic fuzzy pr$e\alpha$-open continuous.

## ***1.Introductiot***

The concept of fuzzy set was introduced by Zadeh in his classical paper [12] in 1965. Using the concept of fuzzy sets, Chang [3] introduced the concept of fuzzy topological space . In [1], Atanassov introduced notion of intuitionistic fuzzy sets in 1986. Using the notion of intuition- istic fuzzy sets, Coker [4] defined the notion of intuitionistic fuzzy topological spaces in 1997. In this paper, we study the classes of normal spaces, namely $\alpha$ -normal spaces and $\pi g\alpha-$ normal spaces in ntuitionistic fuzzy topological spaces, we obtain some properties of these form in intuitionistic fuzzy topological spaces. Moreover, we study the forms of intuitionistic fuzzy $\pi$ generalized $\alpha$ -normality in subspaces, and investigate some of their properties and characterizations.

## ***2.Preliminaries***

**Definition 2.1**[1]

An intuitionistic fuzzy set (IFS in short) *A* in $X$ is an object having

the form $A=\{\langle x,\mu_A(x),\nu_A(x)\rangle / x\in X\}$ where the functions $\mu_A : X\rightarrow[0,1]$ and $\nu_A : X\rightarrow[0,1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\nu_A(x)$) of each element $x\in X$ to the set *A*, respectively, and $0\leq\mu_A(x)+\nu_A(x)\leq 1$ for each $x\in X$. Denote by *IFS* (*X*), the set of all intuitionistic fuzzy sets in *X*.

**Definition 2.2** [1]

Let *A* and *B* be intuitionistic fuzzy sets of the form and $B=\{\langle x,\mu_B(x),\nu_B(x)\rangle / x\in X\}$ $A=\{\langle x,\mu_A(x),\nu_A(x)\rangle / x\in X\}$

(a) $A\subseteq B$ if and only if $\mu_A(x)\leq\mu_B(x)$ and $\nu_A(x)\geq\nu_B(x)$ for all $x\in X$

(b) $A$ =B if and only if $A\subseteq B$ and $B\subseteq A$

(c) $A^c = \{\langle x, \nu_A(x), \mu_A(x)\rangle / x \in X\}$
(d) $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x)\rangle / x \in X\}$
(e) $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x)\rangle / x \in X\}$

The intuitionistic fuzzy sets $0_\sim = \{\langle x, 0, 1\rangle / x \in X\}$ and $1_\sim = \{\langle x, 1, 0\rangle / x \in X\}$ are respectively the empty set and the whole set of $X$.

**Definition 2.3** [4]

An intuitionistic fuzzy topology (IFT in short) on $X$ is a family $\tau$ of IFSs in $X$ satisfying the following axioms:
(i) $0_\sim, 1_\sim \in \tau$
(ii) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$
(iii) $\cup G_i \in \tau$ for any family $\{Gi / i \in J\}$

In this case the pair $(X, \tau)$ is called an intuitionistic fuzzy topological **Definition 2.3** [4]

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In this case the pair $(X, \tau)$ is called an intuitionistic fuzzy topological space (IFTS in short) and any IFS in $\tau$ □is known as an intuitionistic fuzzy open set (IFOS in short) in $X$. The complement $A^c$ of an IFOS $A$ in IFTS $(X, \tau)$ is called an intuitionistic fuzzy closed set (IFCS in short in $X$).

**Definition 2.4**[9]

An IFS $A$ in an IFTS $(X, \tau)$ is said to be an
i) intuitionistic fuzzy α-open set (IFαOS in short) if $A \subseteq \text{int}(cl(\text{int}(A)))$.

ii) intuitionistic fuzzy α-closed set (IFαCS in short) if $cl(\mathrm{int}(cl(A))) \subseteq A$.

The family of all IFCS (resp. IFαCS, IFOS, IFαOS) of an IFTS $(X,\tau)$ is denoted by IFC$(X)$ (resp. IFαC$(X)$, IFO$(X)$, IFαO$(X)$).

**Definition 2.5** [12]

Let $A$ be an IFS in an IFTS $(X,\tau)$. Then

i) $\alpha\,\mathrm{int}(A) = \cup\,\{G/G$ is an IFαOS in $X$ and $G \subseteq A\}$

ii) $\alpha cl(A) = \cap\,\{K/K$ is an IFαCS in $X$ and $A \subseteq K\}$

**Definition 2.6**[11]

An IFS $A$ in an IFTS $(X,\tau)$ is said to be an

i) intuitionistic fuzzy regular closed set (IFRCS in short ) if $A = cl(\mathrm{int}(A))$ ii) intuitionistic fuzzy regular open set(IFROS in short ) if $A = \mathrm{int}(cl(A))$ iii) intuitionistic fuzzy generalized closed set (IFGCS in short ) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and $U$ an (IFROS in short) and $U$ an IFOS in $(X,\tau)$.

**Definition 2.7**[4]

An IFS $A$ in an IFTS $(X,\tau)$ is said to be an

i) The finite union of IF regular open sets is said to be IF π-open .

ii) The complement of IF π- open set is said to be IF π-closed.

**Definition 2.8** [10]

An IFS $A$ in $(X,\tau)$ is said to be an intuitionistic fuzzy $\pi g\alpha$-closed set(IF πGαCS in short) if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is an IF$\pi$OS in $(X,\tau)$.

**Definition 2.9**
**[10]**

An IFS $A$ in $(X,\tau)$ is said to be an intuitionistic fuzzy $\pi g\alpha$-open set (IF πGαOS in short) if the complement $A^C$ is an IFπGαCS in $(X,\tau)$.

**Remark 2.10** [10]

Every IFCS, IF$\alpha$CS, IFRCS, IFGCS is an IFπGαCS, but converses may not true in general.

## Remark 2.11

(i) Every IFπOS in $(X,\tau)$ is an IFOS in $(X,\tau)$. [8]

(ii) Every IFOS in $(X,\tau)$ is an IFαOS in $(X,\tau)$. [4]

(iii) Every IFπOS in $(X,\tau)$ is an IFπGOS. [8]

**3. Intuitionistic Fuzzy $\alpha$- Normal Spaces**

**Definition 3.1**

An intuitionistic fuzzy topological space $(X,\tau)$ is said to be intuition- istic fuzzy $\alpha$-normal space (or in short IF $\alpha$ -N) if for every pair of disjoint intuitionistic fuzzy closed sets $A$, there exist two disjoint intuitionistic fuzzy $\alpha$-open sets (IF$\alpha$OSs) $U$ and $V$ such that $A \subseteq U$, $B \subseteq V$.

**Theorem 3.2**

Let $(X,\tau)$ be an intuitionistic fuzzy topological space the following are equivalent :

1) $X$ is an intuitionistic fuzzy $\alpha$-normal space .

2) For every pair of an intuitionistic fuzzy open sets $U$ and $V$ whose union is $1_\sim$ there exist intuitionistic fuzzy $\alpha$-closed sets $A$ and $B$ such that $A \subseteq U$, $B \subseteq V$ and $A \cup B = 1_\sim$.

3) For every intuitionistic fuzzy closed set $H$ and every intuitionistic fuzzy open $K$ containing $H$, there exists an intuitionistic fuzzy $\alpha$-open set $U$ such that $H \subseteq U \subseteq \alpha - cl(U) \subseteq K$.

4) For every pair of an intuitionistic fuzzy disjoint $\alpha$-closed sets $H$ and $K$ of $X$ there exists an intuitionistic fuzzy $\alpha$-open set $U$ of $X$ such that $H \subseteq U$ and $IF\alpha - cl(U) \cap K = 0_{\sim}$ .

5) For every pair of an intuitionistic fuzzy disjoint $\alpha$-closed sets $H$ and $K$ of $X$ there exists an intuitionistic fuzzy $\alpha$-open sets $U$ and $V$ of $X$ such that $H \subseteq U, K \subseteq V$ and $IF\alpha - cl(U) \cap IF\alpha - cl(V) = 0_{\sim}$

***Proof***

1) $\Rightarrow$ 2)

Let $U$ and $V$ be two intuitionistic fuzzy open sets in an IF $\alpha$-normal space $X$ such that $U \cup V = 1_{\sim}$. Then $U^c, V^c$ are intuitionistic fuzzy disjoint closed sets. Since $X$ is an intuitionistic fuzzy $\alpha$-normal space there exist intuitionistic fuzzy disjoint $\alpha$-open sets $U_1$ and $V_1$ such that $U^c \subseteq U_1$ and $V^c \subseteq V$.Let $A = U^c{}_1, B = V_1^c$ .Then $A$ and $B$ are intuitionistic fuzzy $\alpha$ -closed sets such that $A \subseteq U$, $B \subseteq V$ and $A \cup B = 1_{\sim}$.

2) $\Rightarrow$ 3)

Let $H$ be intuitionistic fuzzy closed set and $K$ be an intuitionistic fuzzy open set containing $H$ .Then $H^c$ and $K$ are intuitionistic fuzzy open sets such that $H^c \cup K = 1_{\sim}$. Then by (2) there exist an intuitionistic fuzzy $\alpha$-closed sets $M_1$ and $M_2$ such that $M_1 \subseteq H^c$ and $M_2 \subseteq K$ and $M_1 \cup M_2 = 1_{\sim}$.Thus, we obtain $H \subseteq M^c{}_1$, $K^c \subseteq M^c{}_2$ $M^c{}_1 \cap M^c{}_2 = 0_{\sim}$.

Let $U = M_1{}^c$ and $V = M_2{}^c$ .Then $U$ and $V$ are intuitionistic fuzzy disjoint $\alpha$- open sets such that $H \subseteq U \subseteq V^c \subseteq K$. As $V^c$ an intuitionistic fuzzy $\alpha$-closed set, we have $H \subseteq U \subseteq \alpha - cl(U) \subseteq K$.

$3) \Rightarrow 4)$

Let $H$ and $K$ be disjoint IF $\alpha$-closed set of $X$. Then $H \subseteq K^c$ where $K^c$ is IF $\alpha$-open. By the part(3), there exist a IF $\alpha$-open subset $U$ of $X$ such that $H \subseteq U \subseteq \alpha - cl(U) \subseteq K^c$. Thus $IF\alpha cl(U) \bigcap K = 0_{\sim}$.

$4) \Rightarrow 5)$

Let $H$ and $K$ be any disjoint IF $\alpha$-closed set of $X$. Then by the part (4) there exist a IF $\alpha$-open set $U$ containing $H$ such that $IF\alpha cl(U) \bigcap K = 0_{\sim}$. Since $IF\alpha cl(U)$ is an IF $\alpha$-closed, then it is IF $\alpha$-closed. Thus $IF\alpha cl(U)$ and $K$ are disjoint IF $\alpha$-closed sets of $X$. Again by the part (4), there exists a IF $\alpha$-open set $V$ in $X$ such that $K \subseteq V$ and $IF\alpha cl(U) \bigcap IF\alpha cl(V) = 0_{\sim}$.

$5) \Rightarrow 1)$

Let $H$ and $K$ be any disjoint IF $\alpha$-closed sets of $X$. Then by the part 5). There exist IF $\alpha$-open sets $U$ and $V$ such that $H \subseteq U, K \subseteq V$, and $IF\alpha cl(U) \bigcap IF\alpha cl(V) = 0_{\sim}$. Therefore, we obtain that $U \bigcap V = 0_{\sim}$. Hence $X$ is IF $\alpha$- normal space.

**Definition 3.3[5]**

An IF function $f:(X,\tau) \to (Y,\sigma)$ is said to be :

1) Intuitionistic fuzzy pre-continuous function if $f^{-1}(B) \in IFPO(X)$ for every $B \in \sigma$.
2) Intuitionistic fuzzy $\alpha$-continuous function $f^{-1}(B) \in IF\alpha O(X)$ for every $B \in \sigma$.

3) Intuitionistic fuzzy $\alpha$-open function (IF$\alpha O$ function for short) if $f(A)$ is an $IF\alpha OS$ in $Y$ for each $IFOS\ A$ in $X$ .

**Definition 3.4**

An IF function $f:(X,\tau)\to(Y,\sigma)$ is said to be:

1) IF pre $\alpha$-open if $f(U)\in IF\alpha O(Y)$ for each $U\in IF\alpha O(X)$.
2) IF pre $\alpha$-closed if $f(U)\in IF\alpha C(Y)$ for each $U\in IF\alpha C(X)$.
3) IF almost $\alpha$-irresolute if for each IF point $x(\alpha,\beta)$ in $X$ and each IF$\alpha$-neighbourhood $V$ of $f(x)$, $\alpha-cl(f^{-1}(V))$ is an IF$\alpha$-neighbourhood of $x(\alpha,\beta)$.

**Theorem 3.5**

A surjective function $f:(X,\tau)\to(Y,\sigma)$ is an intuitionistic fuzzy pre$\alpha$-open continuous almost $\alpha$ -irresolute function from an intuitionistic fuzzy $\alpha$-normal space $(X,\tau)$ onto $(Y,\sigma)$.Then $(Y,\sigma)$ is an intuitionistic fuzzy $\alpha$-normal space.

***Proof***

Let $A$ be an intuitionistic fuzzy closed set of $Y$ and $B$ an intuitionistic fuzzy open set of $Y$ containing $A$.Then since $f$ is continuous $f^{-1}(A)$ and $f^{-1}(B)$ are intuitionistic fuzzy closed (respt. open ) in $X$ such that $f^{-1}(A)$ and $f^{-1}(B)$. Since $X$ is an intuitionistic fuzzy $\alpha$- normal there exists an intuitionistic fuzzy $\alpha$ -open set $U$ in $X$ such that $f^{-1}(A)\subseteq U\subseteq\alpha-cl(U)\subseteq f^{-1}(B)$, by theorem (3.2) $(f^{-1}(A))\subseteq f(U)\subseteq f(\alpha-cl(U))\subseteq f(f^{-1}(B))$. Since $f$ is an intutionistic fuzzy pre $\alpha$-open almost $\alpha$ -irresolute-surjection function, we obtain $A\subseteq f(U)\subseteq\alpha-cl(f(U))\subseteq B$.Then again by theorem (3.2) .The space

$(Y,\sigma)$ is intuitionistic fuzzy $\alpha$-normal space.

**Theorem 3.6**

A function $f:(X,\tau)\to(Y,\sigma)$ is an intuitionistic fuzzy pre $\alpha$-closed function if and only if for each intuitionistic fuzzy set $A$ in $Y$ and for each intuitionistic fuzzy $\alpha$-open set $U$ in $X$ containing $f^{-1}(A)$ there exist an intuitionistic fuzzy $\alpha$-open set $V$ of $Y$ containing $A$ such that $f^{-1}(V)\subseteq U$.

**Theorem 3.7**

Let $f:(X,\tau)\to(Y,\sigma)$ be an intuitionistic fuzzy pre $\alpha$-closed continuous function from an intuitionistic fuzzy $\alpha$-normal space $X$ onto a space $Y$, then $Y$ is an intuitionistic fuzzy $\alpha$-normal space.

***proof***

Let $M_1$ and $M_2$ are intuitionistic fuzzy disjoint closed sets in $Y$ $f^{-1}(M_1)$ and $f^{-1}(M_2)$ are intuitionistic fuzzy closed sets in $X$. Since $X$ is an intuitionistic fuzzy $\alpha$-normal space, there exist disjoint intuitionistic fuzzy $\alpha$-open sets $U$ and $V$ such that $f^{-1}(M_1)\subseteq U$ and $f^{-1}(M_2)\subseteq V$. By theorem(3.6) there exist an intuitionistic fuzzy $\alpha$-open sets $A$ and $B$ such that $M_1\subseteq A$ and $M_2\subseteq B$, $f^{-1}(A)\subseteq U$ and $f^{-1}(B)\subseteq V$. Also $A$ and $B$ are disjoint .Thus $Y$ is an intuitionistic fuzzy $\alpha$-normal space.

**Definition 3.8**

An intuitionistic fuzzy function

$f:(X,\tau)\to(Y,\sigma)$ is said to be $\alpha$ -closed if $f(U)$ is an IF $\alpha$-closed set in $Y$ for each closed set $U$ in $X$ .

**Theorem 3.9**

Let $f:(X,\tau)\to(Y,\sigma)$ be an ntuitionistic fuzzy $\alpha$-closed continuous surjection and $X$ is an intuitionistic fuzzy normal, then $Y$ is $\alpha$-normal space.

***Proof***

Let $A$ and $B$ be an intuitionistic fuzzy disjoint closed sets in $Y$.

Since $f$ is continuous then $f^{-1}(A)$ and $f^{-1}(B)$ are intuitionistic fuzzy disjoint closed sets in $X$. As $X$ is an intuitionistic fuzzy normal, there exist intuitionistic fuzzy disjoint open sets $U$ and $V$ in $X$ such that $f^{-1}(A)\subseteq U$ and $f^{-1}(B)\subseteq V$ .Then there are intuitionistic fuzzy disjoint open sets $G$ and $H$ in $Y$ such that $A\subseteq G$ and $B\subseteq H$ . Since every intuitionistic fuzzy open set is $\alpha$-open, $G$ and $H$ are intuitionistic fuzzy disjoint $\alpha$-open sets containing $A$ and $B$, respectively. Therefore $Y$ is an intuitionistic fuzzy $\alpha$-normal.

**4. Intuitionistic Fuzzy $\pi g\alpha$ - Normal Spaces**

In this section, we introduce the notion of IF $\pi g\alpha$ -normal space and study some of its properties.

**Definition 4.1**

An IF topological space $X$ is said to be IF $\pi g\alpha$ -normal if for every pair of disjoint IF $\pi g\alpha$ -closed subsets $A$ and $B$

of $X$, there exist disjoint IF $\alpha$-open sets $U$ and $V$ of $X$ such that $A \subseteq U$ and $B \subseteq V$.

**Theorem 4.2**

For an intuitionistic fuzzy topological space $(X,\tau)$ the following are equivalent:

(1) $X$ is $\pi g\alpha$-normal.

(2) for any pair of intuitionistic fuzzy disjoint $\pi g\alpha$-open sets $U$ and $V$ of $X$ there exist disjoint $\pi g\alpha$-closed sets $A$ and $B$ of $X$ such that $A \subseteq U$ and $B \subseteq V$ and $U \bigcup V = X$.

(3) for each IF $\pi g\alpha$-closed set $A$ and an IF $\pi g\alpha$-open set $B$ containing $A$ there exists a IF $\alpha$-open set $U$ such that $A \subseteq U \subseteq IF\alpha - cl(U) \subseteq B$.

(4) for any pair of intuitionistic fuzzy disjoint $\pi g\alpha$-closed sets $A$ and $B$ of $X$ there exists a IF $\alpha$-open set $U$ of $X$ such that $A \subseteq U$ and $IF\alpha - cl(U) \bigcap B = 0_{\sim}$.

(5) for any pair of intuitionistic fuzzy disjoint $\pi g\alpha$-closed sets $A$ and $B$ of

$X$ there exists a IF $\alpha$-open sets $U$ and $V$ of $X$ such that $A \subseteq U$ and $B \subseteq V$ $IF\alpha - cl(U) \bigcap IF\alpha - cl(V) = 0_{\sim}$.

***Proof***

$1) \Rightarrow 2)$

Let $U$ and $V$ be two intuitionistic fuzzy $\pi g\alpha$-open sets in an IF $\pi g\alpha$- normal space $X$ such that $U \bigcup V = 1_{\sim}$. Then $U^c, V^c$ are intuitionistic fuzzy disjoint $\pi g\alpha$- closed sets. Since $X$ is an intuitionistic fuzzy $\pi g\alpha$- normal space there exist intuitionistic fuzzy disjoint $\alpha$-open sets $U_1$ and $V_1$ such that $U^c \subseteq U_1$ and

$V^c \subseteq V_1$. Let $A = U_1^{\ c}, B = V_1^{\ c}$ .Then $A$ and $B$ are intuitionistic fuzzy $\alpha$-closed sets such that $A \subseteq U,\ B \subseteq V$ and $A \bigcup B = 1_{\sim}$.

$2) \Rightarrow 3)$

Let $H$ be intuitionistic fuzzy $\pi g\alpha$ - closed set and $K$ be an intuitionistic fuzzy $\pi g\alpha$ -open set containing $H$ .Then $H^c$ and $K$ are intuitionistic fuzzy $\pi g\alpha$ -open sets such that $H^c \bigcup K = 1_{\sim}$ .Then by (2) there exist an intuitionistic fuzzy $\alpha$-closed sets $M_1$ *and* $M_2$ such that $M_1 \subseteq H^c$ and $M_2 \subseteq K$ and $M_1 \bigcup M_2 = 1_{\sim}$ .Thus we obtain $H \subseteq M_1^{\ c},\ K^c \subseteq M_2^{\ c}$ and $M_1^{\ c} \bigcap M_2^{\ c} = 0_{\sim}$.

Let $U = M_1^{\ c}$ and $V = M_2^{\ c}$ .Then $U$ and $V$ are intuitionistic fuzzy disjoint $\alpha$ -open sets such that $H \subseteq U \subseteq V^c \subseteq K$ . As $V^c$ an intuitionistic fuzzy $\alpha$-closed set, we have $H \subseteq U \subseteq \alpha - cl(U) \subseteq K$.

$3) \Rightarrow 4)$

Let $H$ and $K$ be disjoint IF $\pi g\alpha$ -closed set of $X$ . Then $H \subseteq K^c$ where $K^c$ is IF $\pi g\alpha$ -open. By the part(3), there exist a IF $\alpha$-open subset $U$ of $X$ such that $H \subseteq U \subseteq \alpha - cl(U) \subseteq K^c$.Thus $IF\alpha cl(U) \bigcap k = 0_{\sim}$.

$4) \Rightarrow 5)$

Let $H$ and $K$ be any disjoint IF $\pi g\alpha$ -closed set of $X$ .Then by the part (4), there exist a IF $\alpha$-open set $U$ containing $H$ such that $IF\alpha cl(U) \bigcap k = 0_{\sim}$. Since $IF\alpha cl(U)$ is an IF $\alpha$-closed, then it is IF $\pi g\alpha$ -closed . Thus $IF\alpha cl(U)$ and $K$ are disjoint IF $\pi g\alpha$ -closed sets of $X$ . Again by the part (4), there exist a IF$\alpha$ -open set $V$ in $X$ such that $K \subseteq V$ and $IF\alpha cl(U) \bigcap IF\alpha cl(V) = 0_{\sim}$.

$5) \Rightarrow 1)$

Let $H$ and $K$ be any disjoint IF $\pi g\alpha$-closed sets of $X$. Then by the part (5), there exist IF $\alpha$-open sets $U$ and $V$ such that
$H \subseteq U, K \subseteq V$, and
$IF\alpha cl(U) \cap IF\alpha cl(V) = 0_{\sim}$. Therefore we obtain that $U \cap V = 0_{\sim}$.
Hence $X$ is IF $\pi g\alpha$- normal space.

**Lemma 4.3**

a) The image of IF $\alpha$-open subset under an IF- open continuous function is IF $\alpha$-open subset.

b) The image of IF $\alpha$-open subset under an open continuous function is IF $\alpha$-open subset.

**Lemma 4.4**

The image of IF regular open subset under an open and closed continuous function is IF regular open subset.

**Lemma 4.5 [5]**

The image of IF $\alpha$-open subset under IF- open and IF-closed continuous function is IF $\alpha$-open subset.

**Theorem 4.6** If $f : X \to Y$ be an IF-open and IF-closed continuous bijection function and $A$ be a IF $\pi g\alpha$-closed set in $Y$, then $f^{-1}(A)$ is IF $\pi g\alpha$-closed set in $X$.

***Proof***

Let $A$ be an $\pi g\alpha$-closed set in $Y$ and $U$ be any IF $\pi$-open set of $X$ such that $f^{-1}(A) \subseteq U$. Then by lemma (4.5), we have $f(U)$ is IF $\pi$- open set of $Y$ such that $A \subseteq f(U)$. Since $A$ is an IF $\pi g\alpha$-closed set of $Y$ and $f(U)$ is IF $\pi$- open set in $Y$. Thus IF $\alpha cl(A) \subseteq U$

. By lemma (4.3) we obtain that $f^{-1}(A) \subseteq f^{-1}(IF\alpha - cl(A)) \subseteq U$, where $f^{-1}(IF\alpha - cl(A))$ is $\alpha$-closed in $X$. This implies that IF $\alpha - cl(f^{-1}(A)) \subseteq U$. Therefore $f^{-1}(A)$ is IF $\pi g\alpha$-closed set in $X$.

**Theorem 4.7**

If $f : X \to Y$ be an IF-open and IF-closed continuous bijection function and $X$ be a IF $\pi g\alpha$-normal space , then $Y$ is IF $\pi g\alpha$-normal space .

***Proof***

Let $A$ and $B$ be any disjoint $\pi g\alpha$ -closed set in $Y$. Then by theorem (4.6) $f^{-1}(A)$ and $f^{-1}(B)$, are disjoint of IF $\pi g\alpha$-closed set in $X$. By IF $\pi g\alpha$-normality of $X$, there exist IF $\alpha$-open subsets $U$ and $V$ of $X$ such that $f^{-1}(A) \subseteq U$, $f^{-1}(B) \subseteq V$ and $U \cap V = 0_{\sim}$. By assumption, we have $A \subseteq f(U)$, $B \subseteq f(V)$ and $f(U) \cap f(V) = 0_{\sim}$. By lemma (4.3) $f(U)$ and $f(V)$ are disjoint IF $\alpha$-open set of $Y$ such that $A \subseteq f(U)$, $B \subseteq f(V)$. Hence $Y$ is IF $\pi g\alpha$-normal space.

**5. IF $\pi g\alpha$-normality in subspaces**

**Lemma 5.1 [5]**

If $M$ be a IF $\pi$-open subspace of a space $X$ and $U$ be an IF $\pi$-open subset of $X$, then $U \cap M$ is IF $\pi$- open set in $M$.

**Lemma 5.2**

If $A$ is both IF $\pi$-open and IF $\pi g\alpha$-closed subset of a space $X$, then $A$ is an IF $\alpha$-closed set in $X$.

***Proof***

Since $A$ is both IF $\pi$-open and IF $\pi g\alpha$-closed subset of a space $X$ and since $A \subseteq A$, then $IF\,\alpha cl(A) \subseteq A$. But $A \subseteq IF\alpha cl(A)$. Then $A = IF\,\alpha cl(A)$

Hence $A$ is an IF $\alpha$-closed set in $X$.

**corollary 5.3**.

If $A$ is both IF $\pi$-open and IF $\pi g\alpha$-closed subset of a space $X$, then $A$ is an IF $\alpha$ regular-closed set in $X$.

**Theorem 5.4**

Let $M$ be an IF $\pi$-open subspace of a space $X$ and $A \subseteq M$. If $M$ is an IF $\pi g\alpha$-closed subset of a space $X$ and $A$ is an IF $\pi g\alpha$-closed subset of $M$. Then $A$ is an IF $\pi g\alpha$-closed subset of $X$.

**Lemma 5.5**

Let $M$ be an intuitionistic fuzzy closed domain subspace of a space $X$. If $U$ is an IF $\alpha$-open set in $X$, then $U \cap M$ is an IF $\alpha$-open set in $M$.

**Theorem 5.6**

An intuitionistic fuzzy $\pi g\alpha$-closed and IF $\pi$-open subspace of an intuitionistic fuzzy $\pi g\alpha-$normal space is an intuitionistic fuzzy $\pi g\alpha-$normal.

***Proof***

Suppose that $M$ is an IF $\pi g\alpha$-closed and IF $\pi$-open subspace of
an intuitionistic fuzzy $\pi g\alpha-$normal space $X$. Let $A$ and $B$ be any intuitionistic fuzzy disjoint $\pi g\alpha$-closed subsets of $M$. Then by theorem (5.4), we have $A$ and $B$ are intuitionistic fuzzy disjoint $\pi g\alpha$-closed sets in $X$. By intuitionistic fuzzy $\pi g\alpha-$normality of

$X$ , there exist ntuitionistic fuzzy $\alpha$-open subsets $U$ and$V$ of such that $A \subseteq U$, $B \subseteq V$ and $U \cap V = 0_{\sim}$.

By corollary ( 5.3) and lemma (5.5), we obtain that $U \cap M$ and $V \cap M$ are intuitionistic fuzzy disjoint $\alpha$-open sets in $M$ such that $A \subseteq U \cap M$ and $B \subseteq V \cap M$ . Hence, $M$ is an intuitionistic fuzzy $\pi g \alpha -$normal subspace of intuitionistic fuzzy $\pi g \alpha -$normal space $X$ .

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